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DISCRETE MATHEMATICS

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PREFACE

This book is intended for a one-semester or a one-quarter introductory course in discrete mathematics. It is appropriate for any student who has had the equivalent of two years of high school algebra. The recommended computer science prerequisite is one programming course using a higher-level language so that the examples drawn from computer science will be more meaningful.

Courses in discrete mathematics have been recommended for mathematics majors (see [Recommendations, 1981]), for secondary teachers of mathematics (see [Recommendations, 1982]), and for computer science majors (see [Curriculum, 1968, 1979]). The increased interest in discrete mathematics is principally attributable to the rise of computer science; however, discrete mathematics is also important in many other fields, such as operations research, engineering, and economics. Besides its applicability, discrete mathematics provides an ideal framework for sharpening problem-solving skills.

The topics treated in this book, elementary combinatorics (counting methods and graph theory), elementary Boolean algebra, and introductory automata theory, reflect my view of what material should be treated in an introductory course in discrete mathematics. I believe that topics such as monoids, applied group theory, and Polya's theory of enumeration belong in a more advanced course. There is more than enough material in this book for a one-semester or a one-quarter course so that it is possible to tailor the text to the needs of a particular audience. (In two quarters, all of the material can be covered.)

can describe it by listing the elements in it. For example, the equation

$$A = \{1, 2, 3, 4\} \tag{1.1.1}$$

describes a set A made up of the four elements 1, 2, 3, and 4. A set is determined by its elements and not by any particular order in which the elements might be listed. Thus A might just as well be specified as

$$A = \{1, 3, 4, 2\}.$$

The elements making up a set are assumed to be distinct and although for some reason we may have duplicates in our list, only one occurrence of each element is in the set. For this reason we may also describe the set A defined in (1.1.1) as

$$A = \{1, 2, 2, 3, 4\}.$$

If a set is a large finite set or an infinite set, we can describe it by listing a property necessary for membership. For example, the equation

$$B = \{x \mid x \text{ is a positive, even integer}\} \tag{1.1.2}$$

describes the set B made up of all positive, even integers; that is, B consists of the integers 2, 4, 6, and so on. The vertical bar "|" is read "such that." Equation (1.1.2) would be read, "B equals the set of all x such that x is a positive, even integer." Here the property necessary for membership is "is a positive, even integer." Note that the property appears after the vertical bar.

If X is a finite set, we let

$$|X| = \text{number of elements in } X.$$

Given a description of a set X such as (1.1.1) or (1.1.2) and an element x, we can determine whether or not x belongs to X. If the members of X are listed as in (1.1.1), we simply look to see whether or not x appears in the listing. In a description such as (1.1.2), we check to see whether the element x has the property listed. If x is in the set X, we write $x \in X$ and if x is not in X, we write $x \notin X$. For example, if $x = 1$, then $x \in A$, but $x \notin B$, where A and B are given by equations (1.1.1) and (1.1.2).

The set with no elements is called the **empty** (or **null** or **void**) set and is denoted $\varnothing$. Thus $\varnothing = \{\ \}$.

Two sets X and Y are **equal** and we write $X = Y$ if X and Y have the same elements. To put it another way, $X = Y$ if whenever $x \in X$, then $x \in Y$ and whenever $x \in Y$, then $x \in X$.

EXAMPLE 1.1.1. If

$$A = \{x \mid x^2 + x - 6 = 0\}, \qquad B = \{2, -3\},$$

then $A = B$.

Suppose that X and Y are sets. If every element of X is an element of Y, we say that X is a **subset** of Y and write $X \subseteq Y$.

EXAMPLE 1.1.2. If

$$C = \{1, 3\} \qquad \text{and} \qquad A = \{1, 2, 3, 4\},$$

then C is a subset of A.

Any set X is a subset of itself, since any element in X is in X. If X is a subset of Y and X does not equal Y, we say that X is a **proper subset** of Y. The empty set is a subset of every set (see Exercise 43). The set of all subsets (proper or not) of a set X, denoted $\mathcal{P}(X)$, is called the **power set** of X. In Section 2.1 (Example 2.1.2) we will show that if $|X| = n$, then $|\mathcal{P}(X)| = 2^n$.

EXAMPLE 1.1.3. If $A = \{a, b, c\}$, the members of $\mathcal{P}(A)$ are

$$\varnothing, \quad \{a\}, \quad \{b\}, \quad \{c\}, \quad \{a, b\}, \quad \{a, c\}, \quad \{b, c\}, \quad \{a, b, c\}.$$

All but $\{a, b, c\}$ are proper subsets of A. For this example,

$$|A| = 3, \qquad |\mathcal{P}(A)| = 2^3 = 8.$$

Given two sets X and Y, there are various ways to combine X and Y to form a new set. The set

$$X \cup Y = \{x \mid x \in X \text{ or } x \in Y\}$$

is called the **union** of X and Y. The union consists of all elements belonging to either X or Y (or both).

The set

$$X \cap Y = \{x \mid x \in X \text{ and } x \in Y\}$$

is called the **intersection** of X and Y. The intersection consists of all elements belonging to both X and Y.

Sets X and Y are **disjoint** if $X \cap Y = \varnothing$. A collection of sets $\mathcal{S}$ is said to be **pairwise disjoint** if whenever X and Y are distinct sets in $\mathcal{S}$, X and Y are disjoint.

The set

$$X - Y = \{x \mid x \in X \text{ and } x \notin Y\}$$

is called the **difference** (or **relative complement**). The difference $X - Y$ consists of all elements in X that are not in Y.

EXAMPLE 1.1.4. If $A = \{1, 3, 5\}$ and $B = \{4, 5, 6\}$, then

$$A \cup B = \{1, 3, 4, 5, 6\}$$
$$A \cap B = \{5\}$$
$$A - B = \{1, 3\}.$$

The sets $A - B$ and $A \cap B$ are disjoint.